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SOLUTION BY ELIJAH SWIFT, University of Vermont.

The equation, $x^{15} - 1 = 0$, has for its roots

$$r_k = \cos\left(\frac{2k\pi}{15}\right) + i \sin\left(\frac{2k\pi}{15}\right),$$

where $k = 1, 2, 3, \cdots, 15$; $r_k + \frac{1}{r_k} = 2\cos\left(\frac{2\pi k}{15}\right)$. If, then, we divide out all the roots that are not primitive, leaving only $r_1, r_2, r_4, r_7, r_8, r_{11}, r_{13}, r_{14}$, and, noting that $r_{14} = \frac{1}{r_1}, r_{13} = \frac{1}{r_2}, r_{11} = \frac{1}{r_4}, r_8 = \frac{1}{r_7}$, then reduce this equation by the usual method for a reciprocal equation, we shall have the desired equation. This gives us $x^4 - x^3 - 4x^2 + 4x + 1 = 0$.

In a similar manner we can form the equation whose roots are all distinct values of $2\cos\left(\frac{2\pi k}{n}\right)$, where k takes on all integral values, prime to n between and including 1 and $n - 1$.

Also solved by A. M. HARDING and the PROPOSER.

419. Proposed by GEORGE A. OSBORNE, Massachusetts Institute of Technology.

Show that

$$1^5 + 2^5 + 3^5 + \cdots + n^5 + 1^7 + 2^7 + 3^7 + \cdots + n^7 = 2(1 + 2 + 3 + \cdots + n)^4.$$

SOLUTION BY ELMER SCHUYLER, Brooklyn, N. Y.

By induction we must show that

$$(A) \quad \begin{aligned} &1^5 + 2^5 + 3^5 + \cdots + n^5 + (n+1)^5 + 1^7 + 2^7 + 3^7 + \cdots + n^7 \\ &\qquad + (n+1)^7 = 2(1 + 2 + 3 + \cdots + n + (n+1))^4 \end{aligned}$$

Starting with the right member of (A), we have

$$\begin{aligned} &2(1 + 2 + 3 + \cdots + n + \overline{n + 1})^4 \\ &= 2\left[\left\{\frac{n(n+1)}{2}\right\}^4 + 4\left\{\frac{n(n+1)}{2}\right\}^3(n+1) + 6\left\{\frac{n(n+1)}{2}\right\}^2(n+1)^2 \right. \\ &\qquad \left. + 4\left\{\frac{n(n+1)}{2}\right\}(n+1)^3 + (n+1)^4\right] \\ &= 2(1 + 2 + 3 + \cdots + n)^4 + (n+1)^4(n^3 + 3n^2 + 4n + 2) \\ &= 2(1 + 2 + 3 + \cdots + n)^4 + (n+1)^5(n^2 + 2n + 2) \\ &= 2(1 + 2 + 3 + \cdots + n)^4 + (n+1)^5 + (n+1)^7, \end{aligned}$$

which, if the given equation holds, reduces to

$$1^5 + 2^5 + 3^5 + \cdots n^5 + (n+1)^5 + 1^7 + 2^7 + 3^7 + \cdots n^7 + (n+1)^7,$$

and this is the left member of (A). Hence the induction is established.

Also solved by GEORGE Y. SOSNOW, WALTER C. EELLS, J. L. RILEY, CLIFFORD N. MILLS, JAMES A. BULLARD, ELBERT H. CLARKE, A. M. HARDING, R. M. MATHEWS, B. F. YANNEY, ELIJAH SWIFT, and S. A. JOFFE.